Task 0

Take a few minutes and write out how you normally teach subgroups and/or how you were taught subgroups.

Task 1

Definition: A subgroup is a subset of a group that is itself a group with respect to the same operation.

Prove that ($5\boldsymbol{Z}$,+) is a subgroup of ($\boldsymbol{Z}$, +). (By definition $5\boldsymbol{Z}$ is a subset of $\boldsymbol{Z}$.)

1. *Closure*

2. *Identity*

3. *Inverses*

4. *Associativity* *5Z* inherits associativity from *Z*

Task 2

Time to make a conjecture!

According to our definition of subgroup, we must check all four group axioms to ensure a subset is a group (Closure, Identity, Inverses, Associativity).

Come up with a smaller set of conditions that are sufficient (and necessary) to ensure that a subset of a group is a subgroup.

Note: What we want is a weaker set of conditions, so that it will be easier to prove things are subgroups. There is more than one such set of conditions.

Write and prove a theorem based on your conditions.

- If your theorem can be stated as an "if and only if" statement, state it this way.
- If it can not be stated as an "if and only if" give an example to explain why.

Task 3

Suppose that (G, x) is a group, and suppose that H is a subgroup of G.

1. Prove that the identity of H must be the same as the identity of G.

2. Prove that the inverse of any element of H must be the same as that element's inverse in G.

Task 4

Theorem: Let ***G*** be a group and H a subset of ***G***. Then ***H*** is a subgroup iff 1) ***H*** is nonempty, 2) ***H*** is closed under the operation defined on ***G***, and 3) each element of ***H*** has its inverse in ***H***.

Proof:

Task 5

Consider this conjecture: "Any closed subset of a group is a subgroup."

Prove or disprove this conjecture.

Task 6

Consider the following "proof" of the conjecture that closure is sufficient:

"Because the subset is closed, only elements from the subset appear in a row of the table. So each element of the subset must appear in any row. Since symmetry A must appear in the row for symmetry A, the identity must be in the subset. Then since the identity must appear in the row for symmetry A, the inverse of A must be in the subset. Finally, since associativity holds for the whole group it must hold for the subset. Therefore the subset is a group and hence a subgroup."

Where is the flaw in this argument? Are there cases where this proof works? Can this be used to make a new subgroup theorem (different than the one proved in Task 4)?