1) **Generating** student ways of reasoning

*Teachers can do this by:*

* Facilitating student engagement in meaningful tasks and mathematical activity related to an important mathematical point
* Eliciting student reasoning and contributions
* Actively inquiring into student thinking

*Description of High Quality:*

Students were engaged in generalizing their thinking to make mathematical claims that they then tested/supported with arguments, examples, and counterexamples. The teacher tried to connect the task to the students' prior experiences with guidance that helped facilitate continued engagement in the task.

The teacher explicitly asked students to share their approaches to the tasks and their reasoning. The focus was not on what answer they got, but more on how they arrived at that answer, how they were thinking about the problem, the method they took, and a justification for choosing that method.

The teacher engaged in conversations with the students about their thinking. These conversations indicate that the teacher was assessing their own understanding of students' thinking. There are instances of probing, rephrasing of student contributions, and re-presentations. It seems as though the teacher is trying to figure out "how are my students thinking about this?" as opposed to evaluating the correctness of an answer.

2) **Building** on student contributions

*Teachers can do this by:*

* Being responsive to student contributions and use student contributions to inform the lesson
* Guiding and managing the development of the mathematical agenda

*Description of High Quality:*

The instructor listened to and explored student’s contributions when appropriate and utilized these ideas to inform the lesson and follow up questions. Questions about student contributions were asked to the class. The teacher used intermediate student progress to re-direct and focus small group work towards the important mathematical ideas. The instructor often took advantage of student thinking that they felt promoted the mathematical agenda by shifting focus to a student-generated justification. There was a willingness and space to follow/explore unexpected contributions - where the students are doing the exploration.

The teacher monitored small group work and both recorded and used student contributions during whole class discussions as a way to promote the development of the intended mathematical ideas. The teacher guided student explorations towards the mathematical goals of the day. The teacher's questions and interactions appeared to be directed towards a clear mathematical goal. Additionally the instructor managed both the small group work and the whole class discussions.

3) Developing a **Shared** Understanding

*Teachers can do this by:*

* Engage students in one another's thinking

*Description of High Quality:*

The teacher asked students to reflect on the contributions of other students - e.g., “how are you guys making sense of what they are starting to think about here?”. Teacher encouraged students to ask each other questions about alternate approaches and ways of thinking. Additionally the instructor often pointed out ideas other students had while working on the task as a way to support the students progress. The teacher and students share responsibility for the mathematical ideas.

4) **Connecting** to standard mathematical language and notation

*Teachers can do this by:*

* Teachers introduce language and notation when appropriate
* Teachers support formalizing of student ideas/contributions

*Description of High Quality:*

When appropriate, the teacher notated student contributions with formal mathematical language and notation. This was done after the students had a chance to informally develop the mathematical concepts, and when it was likely that the students could make sense of the formal mathematics by connecting it to their previous work.

Student ideas and contributions were formalized. The students were provided with opportunities to make connections between their work and the more formal mathematical/notation (e.g., students are asked to translate and/or interpret formal mathematical nomination utilizing their own generated ideas).