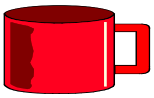
Cooling Coffee



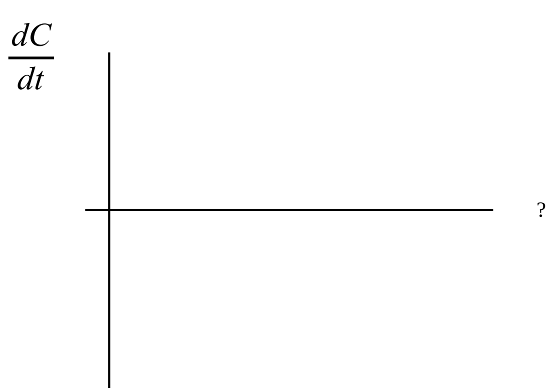
A group of students want to develop a rate of change equation to describe the cooling rate for hot coffee in order that they can make predictions about other cups of cooling coffee. Their idea is to use a temperature probe to collect data on the temperature of the coffee as it changes over time and then to use this data to develop a rate of change equation.

The data they collected is shown in the table below. The temperature C (in degrees Fahrenheit) was recorded every 2 minutes over a 14 minute period.

Time (min)	Temp. (°F)
0	160.3
2	120.4
4	98.1
6	84.8
8	78.5
10	74.4
12	72.1
14	71.5

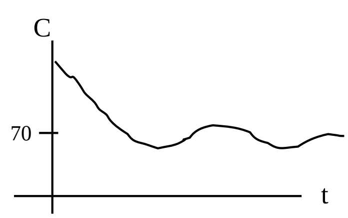
1. Figure out a way to use this data to create a third column whose values approximate $\frac{dC}{dt}$, where C is the temperature of the coffee.

2. Do you expect $\frac{dC}{dt}$ to depend on just the temperature C, on just the time t, or both the temperature C and the time t? Figure out a way to use a graph of your expectation (where $\frac{dC}{dt}$ is on the vertical axis) to help you develop a rate of change equation.



3. One group of students figured out that a reasonable rate of change equation to be $\frac{dC}{dt} = -0.4C + 28$ which they rewrote as $\frac{dC}{dt} = -0.4(C - 70)$. Interpret the meaning of the number 70 in this equation. Does this rate of change equation also make sense for predicting the future temperature of a glass of ice tea? Why or why not?

4. According to the rate of change equation $\frac{dC}{dt} = -0.4(C - 70)$, is it possible for a graph of the **exact** solution to look like the one below? Why or why not?



Population Growth – Limited Resources

A group of biologists want to study the population growth of certain bacteria in a laboratory. The scientists realized that the culture for the bacteria does not provide unlimited resources. Hence, the rate of change equation $\frac{dP}{dt} = kP$ is not appropriate. They conducted experiments to determine how the rate of change of population depends on just the population. The data they collected is shown in the table below (numbers are properly scaled). At various population levels, the scientists measured the population after one day.

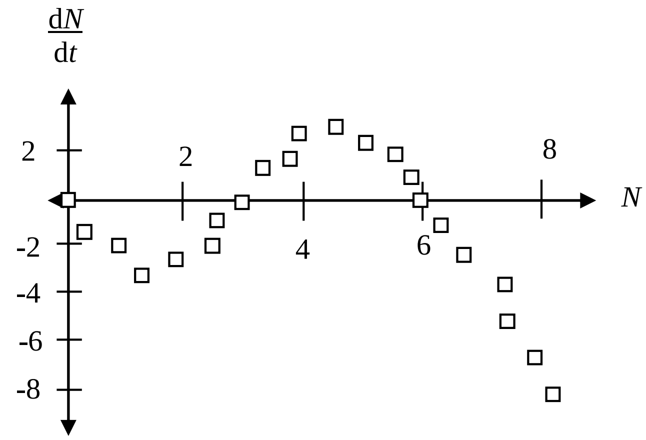
Beginning Population	Population after one day
2	2.34
4	4.54
6	6.62
8	8.58
10	10.40
12	12.10
14	13.66
16	15.10
18	16.42
20	17.60

1. Create a third column whose values approximate $\frac{dP}{dt}$. Explain why the method you used to create this column makes sense.

2. Create a graph of your $\frac{dP}{dt}$ vs. P data and figure out a way to analyze this graph to determine the long term behavior for each of the beginning populations given in the table above.

Analyzing Graphs of Autonmous Differential Equations

1. A group of biologists is studying a particular bug population in a rainforest. They gathered data about these bugs for different population values, N, at different times, t. The scientists reasoned that the rate of change depended only on the population and not on time. They approximated the derivatives $\frac{dN}{dt}$ (as was done with the cooling coffee from before) and plotted $\frac{dN}{dt}$ versus N, as seen below:

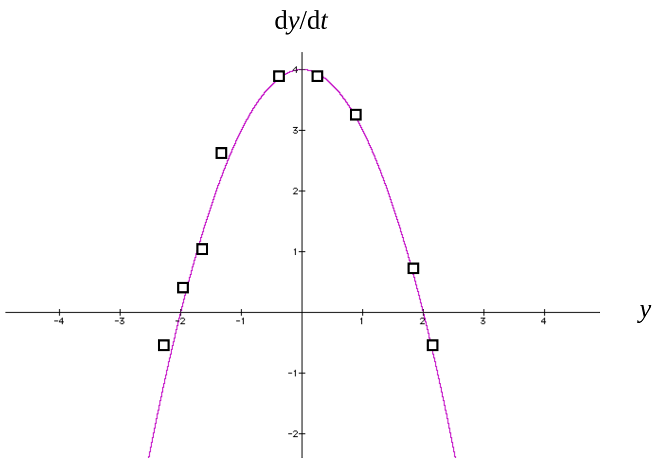


Graph of dN/dt vs. N

For each part below, use the dN/dt vs. N graph to predict what the ultimate fate of the population will be. Describe (in words) the long-term behavior of each solution corresponding to the given initial condition. In addition, illustrate your conclusions with a suitable graph or graphs and classify all equilibrium solutions as either an attractor, repellor, or node.

a) $N(0) = 2$
b) $N(0) = 3$
c) $N(0) = 4$
d) $N(0) = 4.5$
e) $N(0) = 6$
f) $N(0) = 8$

2. Below is a graph of an autonomous differential equation. Figure out the long-term behavior of <u>every</u> possible solution function and illustrate your conclusions with a suitable graph or graphs.

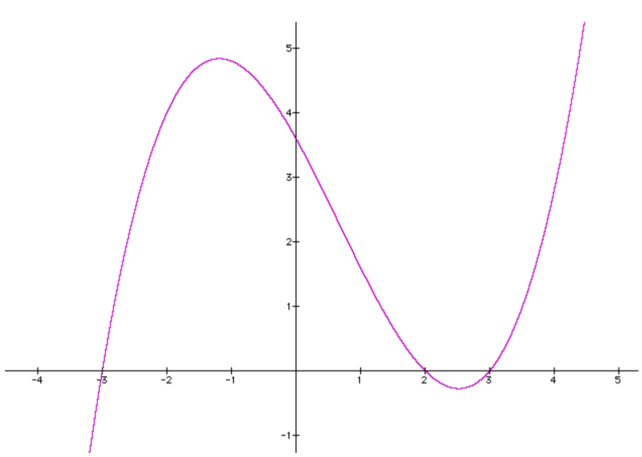


HOMEWORK SET 7

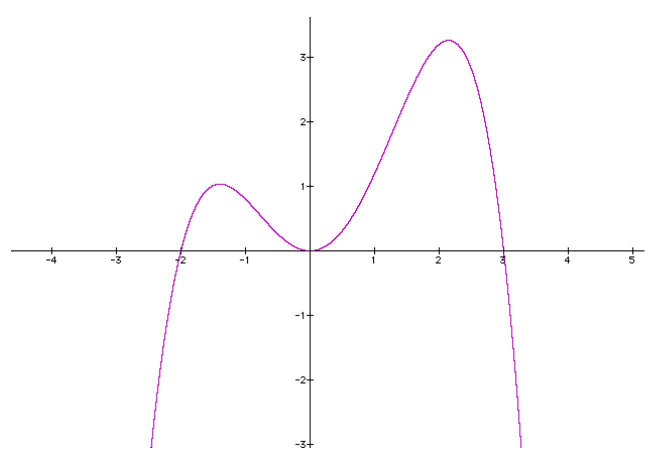
1. Fo this problem, use the coffee cooling rate of change equation $\frac{dC}{dt} = -0.4C + 28$.
   a) Is there ever a time when two cups of coffee, one at initially 160°F and one at 180 °F, are the exact same temperature? Answer this question according to the uniqueness theorem. Comment on whether your answer matches what you expect to happen in real life?
   b) How long will it take a cup of hot coffee that is initially 180 °F to cool down to 100° F? Use the integrating factor technique to figure this out and then check the reasonableness of your answer with Euler's method.

2. For each part below you are provided with a graph of an autonomous differential equation. Figure out the long-term behavior of every possible solution function. Illustrate your conclusions with representative $y(t)$ solution graphs and summarize your findings about the long-term behavior of different solutions in paragraph form.

(a)



(b)



3. For each part in problem 2, create a phase line and classify each equilibrium solution as either an attractor, repellor, or node.

4. Given an autonomous differential equation $\frac{dy}{dt} = f(y)$, give a general strategy for how to use graph of $\frac{dy}{dt}$ vs. y to determine the long term behavior of solution functions.

5. For part b) in problem 2, use the uniqueness theorem to determine if any of the non constant solution functions ever reach the equilibrium solution of $y(t) = 0$ in a finite amount of time.

6. Suppose you wish to predict future values of some quantity, y, using an autonmous differential equation (that is, dy/dt depends explicitly only on y). Experiments have been performed that give the following information:

   - The only equilibrium solutions are $y(t) = 0$, $y(t) = 15$, and $y(t) = 60$
   - If the value of y is 100, the quantity decreases
   - If the value of y is 30, the quantity increases
   - If the value of y is negative, the quantity increases

   a) How many different phase lines match the above? Sketch all possible phase lines.
   b) Provide a rough sketch dy/dt vs. y for each of your phase lines in part a).
   c) For each of your different sketches in part b), develop a differential equation that fits the basic features.

7. In what ways is the letter y in the differential equation dy/dt = .3y both a variable and a function? In what ways is dy/dt a function?