Fish Harvesting

A mathematician at a fish hatchery has been using the differential equation $\frac{dP}{dt} = 2P(1 - \frac{P}{25})$ as a model for predicting the number of fish that a hatchery can expect to find in their pond.

1. Use a graph of dP/dt vs. P, a phase line, and a slope field to analyze what this differential equation predicts for future fish populations for a range of initial conditions. Present all three of these representations and describe in a few sentences how to interpret them.

2. Recently, the hatchery was bought out by fish.net and the new owners are planning to allow the public to catch fish at the hatchery (for a fee of course). This means that the previous differential equation used to predict future fish populations needs to be modified to reflect this new plan. For the sake of simplicity, assume that this new plan can be taken into consideration by including a constant, annual harvesting rate k into the previous differential equation. Which of the three modified differential equations makes the most sense to you and why?

   a) $\frac{dP}{dt} = 2P(1 - \frac{P}{25}) - kP$ b) $\frac{dP}{dt} = 2P(1 - \frac{P - k}{25})$ c) $\frac{dP}{dt} = 2P(1 - \frac{P}{25}) - k$

3. Using the modified differential equation agreed upon from the previous problem, prepare a **one page** report for the new owners that illustrates the implications that various choices of k will have on future fish populations. Your report may include one or more graphical representations but must synthesize your analysis of the effect of different k values in a concise way.

HOMEWORK SET 8

1. For each of the following, develop a report that illustrates (with a suitable graph or graphs) and describes (in words) the way in which the solutions change as the value of *r* changes. Identify the precise value(s) of *r* for which there is a either a change in the number of equilibrium solution(s) or a change in the type of equilibrium solution(s).

   a) $\frac{dy}{dt} = (y - 3)^2 + r$

   b) $\frac{dy}{dt} = y^2 - ry + 1$

   c) $\frac{dy}{dt} = ry + y^3$

   d) $\frac{dy}{dt} = y^6 - 2y^4 + r$

2. For part a) in problem 1, sketch a graph of the equilibirum solutions as *r* varies. Such a graph is referred to as "bifurcation diagram."

3. For part b) in problem 1, sketch a graph of the equilibirum solutions as *r* varies. Such a graph is referred to as "bifurcation diagram."